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THE DYNAMIC PERFORMANCE OF NARROW ACTIVELY TILTING VEHICLES.

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Abstract.

The major advantage of a motorcycle with respect to a passenger car is the possibility of the driver to tilt during cornering and thereby maintaining stability whereas a passenger car will suffer from the risk of capsizing at high speed.
This observation has motivated recently various manufacturers to develop new concepts of narrow vehicles that are able to tilt like a motorcycle but offer the comfort of a passenger car, allowing further downsizing of vehicles enabling higher fuel efficiency. As a result, there seems to arise a whole new class of vehicles, which cannot be compared, to either passenger cars or motorcycles, with typical examples such as the Ford Gyron and the GM Lean Machine. More recent examples are the Daimler Benz Life Jet and the Carver of Brink Dynamics.
This paper includes a mathematical treatment of the dynamic behaviour of such vehicles, denoted here as a Narrow Tilting Vehicle (NTV). The analysis is based on a vehicle concept consisting of three main rigid bodies, a front assembly (front fork), a mid-assembly (passenger cabin) and a two-wheeled rear-assembly (engine, rear part of chassis including rear suspension, wheels, etc.).
Important criteria for the designer are a stable behaviour and, on the other hand, a smooth, responsive cornering performance. As it turns out, various eigenmodes related to yaw, steering and roll behaviour of the vehicle are working against each other in this respect. Obviously, many vehicle design parameters play an important role here which have a major impact on the vehicle behaviour. One might distinguish the overall dimensions, mass distribution, front-fork geometry, tyre characteristics, suspension layout, the tilting controller design, etc. Due to the large variety in parameter-combinations, sensitivity regarding one single parameter always depends on the selected reference case for the remaining parameters, and therefore cannot be extrapolated easily to reach its optimal value.
In the paper, the design parameters having a major impact on the vehicle performance are identified. The impact of parameter modifications on the vehicle stability and handling performance is discussed on the basis of mathematical analysis, and interpreted in physical terms (supported by numerical analysis and experimental results). That means that, finally, tendencies in changes in vehicle performance can be related to classes of vehicle parameters (i.e. having a similar impact) allowing a better understanding of the relationships between vehicle performance and design, and resulting in general guidelines for the design of this new class of vehicles.

Introduction

This paper describes a mathematical treatment of the behaviour of a special representative of a new class of vehicles, the Narrow Tilting Vehicles (NTV). A Narrow Tilting Vehicle can be described as a vehicle with at least three wheels and with part of the vehicle, usually the cockpit housing the driver and the possible passenger, actively tilting inside a bend during cornering. That means that the driver steering action is transferred in some way to a tilting torque moving the cockpit and bringing it to a steady state situation just like a motorcycle. As a result, moving from transient to steady state, the required tilting torque is reduced and likewise the steering effort (in terms of steering input torque) by the driver.
In a recent paper by Pauwelussen, 1999, a specific representative, the MWV (Man Wide Vehicle) by Brink Dynamics (now known as the Carver) was discussed with main focus on the steady state cornering with some comments on the response of the vehicle on a ramp steer torque driver input. In that paper, the hydraulic tilting mechanism was described in detail. The driver steering input torque had to balance both the tilting torque plus a torsional stiffness connecting the cockpit with the rear assembly of the vehicle, and a front wheel steering torque plus an anti-powersteering to initiate cornering from straight-on driving.
The parameters describing this tilting mechanism were discussed with respect to the stationary vehicle behaviour. In addition, the sensitivity of the vehicle performance was investigated regarding changes of the vehicle design. Major effects were observed for varying torsional stiffness as well as for the antisteer control. It was observed that antisteer control had no effect on driver feedback but merely reduced the required driver input for the same steady state cornering behaviour. Quite the opposite was observed for the stiffness of the torsion bar.
Important vehicle parameters effecting the cornering behaviour were found to be the position of the mass

centre, the tyre characteristics and the additional weight of one passenger.

As a next step, this paper will treat the dynamic equations of an NTV. The vehicle consists of three assemblies, a front-fork , a mid-assembly referred to as the cockpit, and finally the rear assembly. Since this vehicle is very much similar to a motorcycle, the equation are expected to be close to equations describing lateral-directional motorcycle dynamics, see for example Sharp, 1971 and Weir et.al., 1978. There are however some interesting differences. Clearly, the tilting parts (cockpit plus front-fork) are rotating against the nontilting rear assembly. That means that the internal tilting torque must be counteracted by the rear tyre forces which might lead to toe-in effects and additional rear-wheel steering, both having a significant effect on the yaw mode of motion. As a consequence, the rear suspension design has a major impact on the vehicle performance and must be designed with great care.
A second difference is the fact that the tilting axis does not necessarily coincide with the line connecting the front and (averaged) rear tyre-road contacts. The tilting axis might be located above or below the groundsurface, with a possibly nonzero inclination angle, leading to additional interactions between roll- and steering motions.
In order to obtain an understanding of this Narrow Tilting Vehicle, the set of equations, extended with the equations describing the titling mechanism is investigated in the Laplace domain through root locus plots and Bode plots. Assuming linear behaviour and neglecting frame-flexibility, a minimum of six equations is required to describe the NTV, from which principal modes appear including sets of oscillating modes. These last modes include all degrees of freedom with however one dominant one, the yaw rate (a type of weave mode) or the steering angle (a type of wobble mode). The impact of varying vehicle parameters on eigenfrequencies and dampingvalues will be discussed.
Next, the time history behaviour of the vehicle for a step input in the steering torque will be examined, in spite of the fact that this a an unrealistic driver input. To initiate cornering, a large tilting torque is required which can be relaxed once a steady state cornering condition is observed. That means that a driver would normally first apply a steering torque exceeding the one that is required for the intended steady state curve, followed by a lower value. Finally, the results are discussed.

The model equations

We follow the approach by Pauwelussen (1999) and refer to figure 1. The behaviour of the NTV can be described with four degrees of freedom:

- v : lateral speed
- r : yaw rate
- φ : tilting angle of the cockpit
- δ : steering angle

In order to describe a system like the NTV, local vector bases are introduced for each of the rigid bodies comprising the vehicle. All of the equations are written in terms of the local co-ordinate system (x,y,z) attached to the rear-assembly with origin located just below the

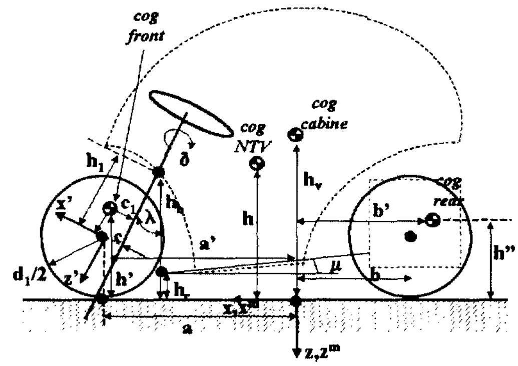

Figure 1.: Model layout

mass centre of the cockpit at ground level.. Transformation matrices are therefore introduced transforming the front-fork and cockpit orientations to these rear-assembly co-ordinates. A local triad $(\underline{b}_x, \underline{b}_y, \underline{b}_z)$ is fixed to the rear-assembly at ground level with $\underline{b}_x$ pointing forward along the vehicle and $\underline{b}_z$ pointing downwards. The origin O' of the front-fork vector basis coincides with the centre of the front wheel hub. The origin O_m of the mid-assembly vector basis coincides with the global origin O. The local unit vectors in z-direction (z' and z^m) in front- and mid-assembly are pointing along the front fork hinge line downwards and vertically downwards within the cockpit symmetry plane, respectively. Initially, all y-directions are starboard.

The following **assumptions** are made:

- All relationships (including the tyre behaviour) are taken linear. Vehicle rotations (roll, pitch, yaw) and their derivatives are assumed to be small.
- Only small perturbations around a constant forward velocity U_0 are considered. i.e. the global velocity vector can be written as $(U_0 + u, v, w)$ with u,v,w small.

- Vertical flexibility of rear-wheels and rear suspension is not taken into account.
- The contributions of rear wheels and engine to the angular momentum of the rear part of the vehicle are neglected. These contributions are assumed to be small compared to the total contribution of the rear-part mass.
- Aerodynamic forces and moments are neglected.
- Reaction of steering torque to the mid-part of the TNV is neglected.

Four equations of motion can be derived for the four degrees of freedom, indicated with index i=1,...,4. With external force and moment $\underline{F}$ and $\underline{M}$, linear momentum $\underline{P}$, angular momentum around the mass centre $\underline{D}$, angular velocity $\underline{\omega}$ of the vector basis and inertial velocity of the mass centre $\underline{V}$, the equations of motion for each of the component bodies with mass m can be written as:

(1) $$\underline{F} = \left(\frac{d}{dt}\underline{P}\right)_{inertial} = m\left(\frac{d}{dt}\underline{V}\right)_{body} + m\underline{\omega}\times\underline{V}$$

(2) $$\underline{M} = \left(\frac{d}{dt}\underline{D}\right)_{inertial} = \left(\frac{d}{dt}\underline{D}\right)_{body} + \underline{\omega}\times\underline{D}$$

Summing up the contributions to the lateral force acting on the total vehicle cancels out all internal forces and leads to the 'Side Force Equation' (i=1) for the NTV. In the same way, summing up the separate expressions of type (2) and taking the component in the z-direction leads to the "Yaw-Equation' (i=2). Application of (2) for the front-fork in the local z'-direction (i.e. about the local steering axis for the front-fork) leads to the 'Steering Equation' (i=3). Application of (2) for the front-fork and cockpit in the local x^m – direction (i.e. about the local tilting axis) yields the 'Roll-Equation' (i=4). As a result, four linear equations are derived which can be expressed as follows:

(3) $$\underline{a}_i.\ddot{\underline{x}} + \underline{b}_i.\dot{\underline{x}} + \underline{c}_i.\underline{x} = f_i$$

with $\underline{x} = (v, r, \varphi, \delta)^T$ and f_i the scalars describing the external loading to the full vehicle (i=1,2) or part (i=3,4) of the NTV. Only for the steering angle δ and the tilting angle φ, second order derivatives arise in these equations, i.e. $\underline{a}_i = (0,0,a_{i3},a_{i4})^T$.

To limit the size of this paper, we discuss these equations only briefly, see Annex A and Annex B. The left-hand sides of (3) mainly consist of the principal inertia terms including the steady state yaw rate contributions to the vehicle lateral acceleration plus additional contributions due to rotational accelerations around the tilting axis and steering axis (hinge line) of the NTV. Inertial couplings as well as gyroscopic effects from the front wheel rotation have been incorporated. As far as the loading is concerned, these terms are mainly due to tyre forces as a result of front and rear sideslip and camber, gravitational forces and actively applied tilting and steering torques, T_φ and T_δ related to the driver input torque T_i.

We close this section with some remarks regarding the rear suspension characteristics and load transfer. The tilting torque acts against the rear part of the vehicle and should be balanced by the rear tyre forces. As a result, load transfer occurs, effecting the effective cornering stiffness $C_{\alpha 2}$ and therefore the lateral vehicle characteristics. This effect has been included in the model. In addition, roll-steer effects might be accounted for, depending on the specific rear suspension design. As we shall see later, a low frequency yawmode arises, depending significantly on these effects. Some sensitivity results will be shown in the subsequent sections.

The Tilting Mechanism

As explained by Pauwelussen (1999), a driver-input torque can be transferred to a tilting torque between the cockpit and the rear assembly, and a steering torque to initiate a steering angle at the front wheel. In addition, a flexible connection may exist between the tilting mid-assembly and the vehicle rear frame being proportional to the tiling angle φ. Finally the tilting control device might be equipped with an antisteer control. In case of cornering, a significant high tilting torque is required to overcome the inertia of the mid-assembly. An opposite power steering initiates already the tilting activity, and reduces both the required driver input torque as well as the tilting torque. An active tilting mechanism will, in general, lead to two relationships (linearised in this paper) describing the tilting and steering torques T_φ and T_δ to the driver input torque T_i. Without going too much in detail (see Pauwelussen, 1999), we introduce the following relationships:

(4) $$T_\varphi = [k_1.T_i - k_2.\varphi] - k_3.\dot{\varphi}$$

(5) $$T_\delta = k_4.T_i - A_0[k_1.T_{in} - k_2.\varphi]$$

In (4), the term between square brackets should be interpreted as a balance between tilting torque and torsional flexibility. This is fed back to the steering torque as an antisteer torque cf. (5). Additional tilting damping has been taken into account through the coefficient k_3. Together with (3), we now have a set of six equations for six unknowns (four degrees of freedom and two torques), that can be solved.

Stability behaviour

Let us first examine the stability behaviour. Solving the characteristic equation leads to six eigenvalues, including two complex conjugate pair of poles. These eigenvalues (poles) are shown as root locus plot in figure 2 for vehicle speed between 10 and 50 m/s. Oscillating modes appear to exist for resonant frequencies ω around 10 and 100 rad/sec (1.7 and 15.9 Hz respectively). Figure 3 is a blow up of figure 2 showing the low frequency oscillating mode in more detail.

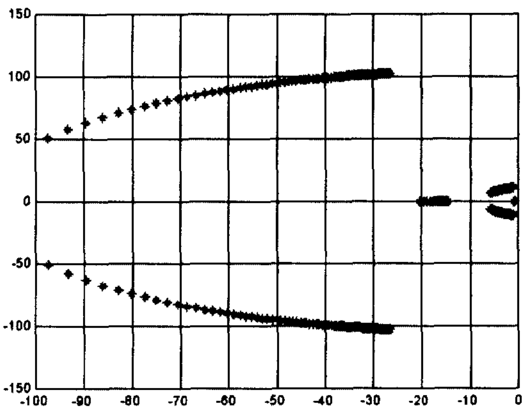

Figure 2.: Root Locus Plot NTV

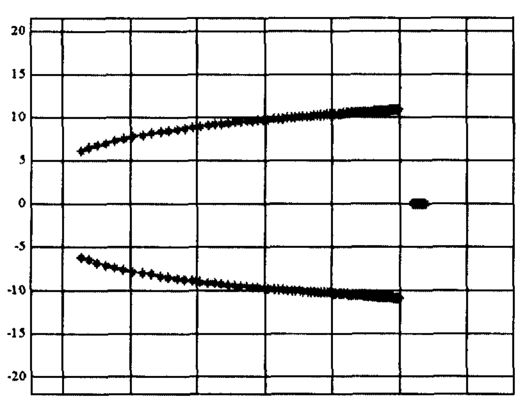

Figure 3.: Root Locus Plot, detail

The low frequency mode shows a limited absolute stability as well as a limited damping ratio (cosine of the position angle of the poles with the real negative axis). It corresponds to a behaviour of the vehicle with a dominant yaw rate and lateral velocity being in phase difference of 90°, and with the steering angle in phase with the lateral velocity. That means that the vehicle is in a kind of fish-tail motion, and we'll refer to it as the *Yaw Mode*. The high frequency oscillating mode shows a yaw rate and lateral velocity being almost in phase, with a 90° phase difference with a dominant steering angle behaviour. We'll therefore refer to this second mode as the *Steering Mode*. The fourth degree of freedom, the tilting angle, only plays a minor role in these modes. The relative amplitudes and phase for the three degrees of freedom (except for φ, and not with the same scale) for these two modes are shown in figure 4.

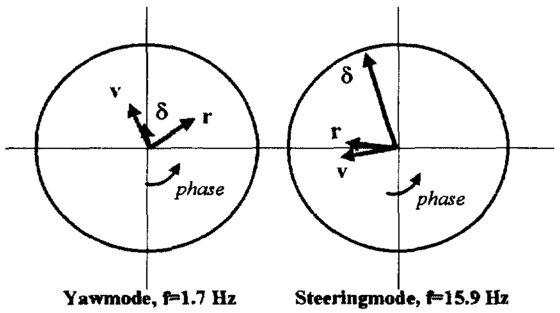

Figure 4.: Yaw- and Steering mode

We have further investigated the sensitivity of the vehicle stability regarding changes in vehicle design parameters as well as in the parameters characterising the tilting system (cf. equations (4) and (5)). Only those parameters are mentioned here with a significant impact in resonant frequency and/or damping ratio for either the Yaw Mode or the Steering Mode. It is to be expected that parameters like masses, tilting axis orientation, positions of mass centres, tyre cornering stiffnesses and camber stiffness are the major contributions to the Yaw Mode. The Steering Mode is mainly effected by parameters related to the front fork such as rake angle, cornering stiffness front, distance between front mass centre and steering axis, and the steering damping. To illustrate these sensitivities, we'll treat the impact of changing the tilting inclination angle μ while maintaining the height of the tilting axle at the connection with the rear assembly at the same level. With positive angle, the steering axis is pointing downward (from aft to for). That means that under cornering conditions, for increasing μ, the front wheel is forced to move more inside the curve reducing the tyre forces. Hence, it corresponds to more understeer or less oversteer which stabilises the vehicle. On the other hand, the tilting inertia of the cockpit plus front fork is increased, leading to lower eigenfrequencies and reduced damping, i.e. corresponding to a destabilising effect. For a tilting axis close to pointing through the cockpit mass centre, the first effect is expected to be dominant. For a tilting axis close to pointing through the front wheel contact point, the second effect is expected to be dominant. Starting from a reference vehicle with μ=0, the effect on the pole connected to the Yaw Mode is illustrated in figure 5 for a vehicle speed of 120 km/h.

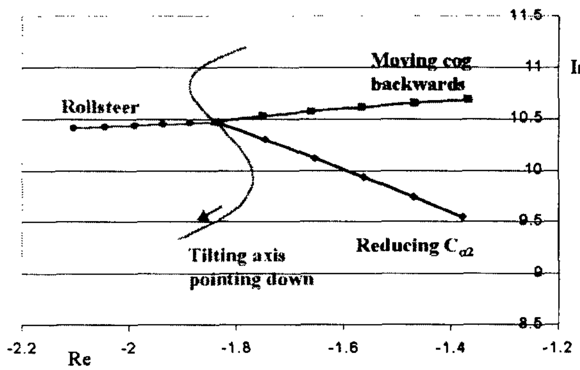

Figure 5.: Sensitivity of root locus Yaw-Mode

This figure shows the destabilising effect with the tilting axis pointing downward within a certain range, due to increasing inertia. The resonant frequency is shown to reduce with increasing μ. For large absolute value of μ, the understeer effect becomes the dominant factor.
In the same figure, the impact of rollsteer (rearward steering due to load transfer), lower cornering stiffness at the rear axle, and horizontally moving the vehicle mass centre is shown. Rollsteer and moving the mass centre forward will increase the stability without effecting the resonant frequency significantly. Variation in cornering stiffness effects both the damped natural frequency and the absolute stability.

Similar analyses can be carried out for the Steering Mode. Moving for example the mass centre of the front fork in a rearward direction with respect to the steering axis, both the resonant frequency as well as the damping of this mode is expected to decrease. Similarly, reducing the cornering stiffness of the front tyre (less resistance against steering) and reduction of the rake angle will both result in lower frequency and lower damping. This is confirmed in figure 6, against a reference situation with a frequency of about 15.9 Hz and a damping ratio ζ of 0.34.

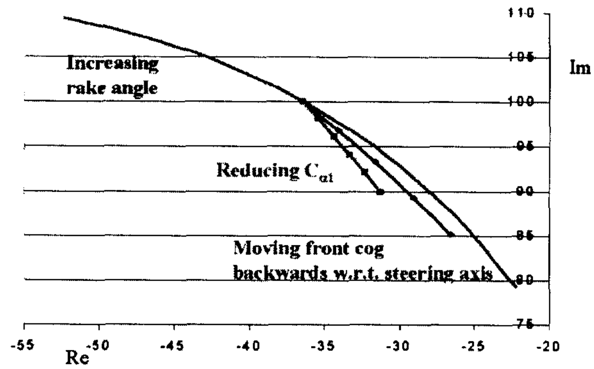

Figure 6.: Sensitivity of root locus Steering-Mode

We finally have determined Bode plots for forward speed U_0 = 120 km/h showing the transfer from a driver controlled input torque T_i to lateral speed v, yaw rate r, tilting angle φ. steering angle δ.

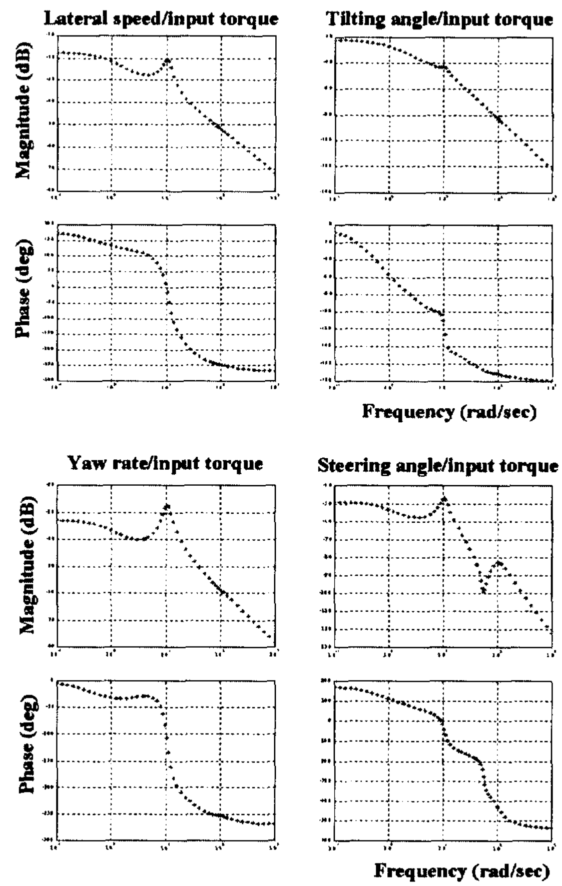

Figure 7.: Frequency transfer functions

One observes the resonant frequency at about 10 rad/sec with only a limited peak for the tilting angle, confirming that rolling is not much affected by yawing. At this frequency, the tilting angle has a phase difference of 90° with the yaw rate, due to the fish-tail type of behaviour of this mode. For low frequency and low vehicle speed, the lateral speed is in phase with the input torque. For high speed (including 120 km/h), the lateral speed is in antiphase with T_i. The resonant frequency of about 100 rad/sec. is only observed for the steering angle transfer.

Smooth cornering of the NTV

The response of the NTV has been determined for a step input in the driver input torque such that a lateral acceleration is reached of 4 m/s^2. Steady state cornering behaviour has been discussed by Pauwelussen, 1999. The transient response for a step in the input torque is not the same as defined in ISO 7401, where an input steering angle is prescribed. In order to reach such a step-input in the steering angle, the driver input torque needs to exceed the steady state value at the start of the transient manoeuvre (to overcome the inertia of the cockpit). Calculations have been carried out for 80 km/h.

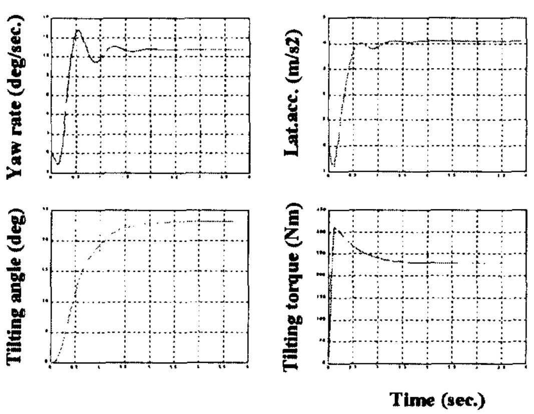

Figure 8.: Transient response

The tilting damping has been omitted in these calculations, being proportional to the rate of the tilting angle. One observes a quick response in the tilting torque with a local overshoot to initiate the tilting of the cockpit. With tilting damping present, this overshoot would show a steeper descent beyond this first maximum. The yaw rate starts with a small delay of about 0.2 sec. and then rises quickly to a significant level. Some oscillations are observed due to the low damping of the Yaw Mode. Similar but smaller oscillations are observed in the lateral acceleration versus time, behaving only slightly delayed with respect to the yaw rate. The small delay times will result in a good subjective driver assessment of the NTV handling performance.

We have examined to what extent the anti-steer control in the tilting system effects the transient behaviour of the TNV. It appears that the response in yaw rate and lateral acceleration is slightly effected (risetime is increasing) but the main affect is in the tilting torque, being required to roll the cockpit. Without antisteer control, this tilting torque is increased with more then 30 %. That means that more energy is required for this transient behaviour. In other words, the antisteer control serves to derive a better design of the tilting control with lower weights and power consumption.

DISCUSSION

The dynamic behaviour of a class of vehicles has been investigated. These vehicles, denoted as Narrow Tilting Vehicles, tilt like a motorcycle during cornering but cannot be compared to motorcycles. Only linear behaviour has been discussed, neglecting frameflexibility, saturation of tyre behaviour, rearside suspension etc. That means that the results of this paper should only be considered in qualitative terms, where more extended multi-body analyses would lead to more realistic quantitative behaviour.

The stability of the vehicle has been studied, with emphasis on two modes of motion, related to yaw motion and front fork steering stability. Especially the first is observed to have low damping, and the impact of vehicle parameters on the stability properties has been treated. Parameters like the tilting axis orientation and the roll-steer characteristics of the rear part of the vehicle appear to have a significant effect on the vehicle yaw stability.

Considering the vehicle behaviour in time, typical reference manoeuvres might be investigated. The steady state behaviour has been discussed earlier. The transient performance shows a rather quick response to changes in driver input torque, compared to passenger cars.

The limited size of this paper only allows presentation of some of the results that were obtained from the linearised model. More results will be reported by the author in subsequent publications.

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Annex A.: Notation

a' : horizontal distance cockpit cog to front fork cog
b' : horizontal distance cockpit cog to rear cog
a : horizontal distance cockpit cog to front fork origin
b : horizontal distance cockpit cog to rear origin
B : Fork damping
c_1 : Distance (along x') between front-fork steering axis and front fork cog.
$C_{\alpha 1}$: Cornering stiffness front tyre
$C_{\alpha 2}$: Equivalent cornering stiffness rear tyres
C_γ : Camber stiffness front tyre
F_{z1} : Front tyre load
g : Acceleration of gravity
h_m : Distance of cockpit mass centre to tilting axis
h_f : Distance of front fork mass centre to tilting axis
h_t : Distance of front tyre contact point to tilting axis
$I_{f,zz}$: Local front fork moment of inertia in z'-direction
$I_{mf,xx}$: Sum of front fork and cockpit moments of inertia (local mid-assembly co-ordinates)
I_{zz} : Global moment of inertia in z-direction
k : Front wheel trail
m : Total mass vehicle
m_f : Mass front fork
m_m : Mass cockpit including driver
r : yaw rate
T_i : Driver input steering torque
T_δ : Front wheel steering torque
T_φ : Tilting torque
U_0 : Reference forward velocity
v : Lateral velocity
α_1 : Slip angle front wheel
α_2 : Slip angle rear axle
γ : Camber angle front
δ : Steering angle front wheel
μ : Tilting axis inclination angle
φ : Tilting angle cockpit against rear assembly

Annex B.: Equations of Motion

The equations of motion are described by

$$\underline{a}_i.\ddot{\underline{x}} + \underline{b}_i.\dot{\underline{x}} + \underline{c}_i.\underline{x} = f_i$$

with the right-hand and left-hand side described in tables 1 and 2, below.

Table 1. System equations

Equation	$\underline{a}_i.\ddot{\underline{x}} + \underline{b}_i.\dot{\underline{x}} + \underline{c}_i.\underline{x}$
Side Force	$m.(\dot{v} + U_0.r)$ + contributions due to rotational accelerations $\dot{r}$, $\ddot{\varphi}$ and $\ddot{\delta}$
Yaw	$I_{zz}.\dot{r} + (a'.m_f - b'.m_r).(\dot{v} + U_0.r)$ + inertia contributions from $\ddot{\varphi}$ (local xz-coupling) and $\ddot{\delta}$
Roll	$\ddot{\varphi}.\left(I_{mf,xx} + m_m.h_m^2 + m_f.h_f^2\right) + \left(m_m.h_m + m_f.h_f\right)(\dot{v} + r.U_0) +$ inertia contributions from $\dot{r}$ and $\ddot{\delta}$ (local xz-coupling) and front wheel rotation (gyroscopic effects)
Steering	$\ddot{\delta}.(I_{f,zz} + m_f.c_1^2) + m_f.c_1.(\dot{v} + U_0.r)$ + contributions from from $\dot{r}$ and $\ddot{\varphi}$ (local xz-coupling) and front wheel rotation.

Table 2. The loading vector

Equation	Loading f_i
Side Force	$C_{\alpha 1}.\alpha_1 + C_{\alpha 2}.\alpha_2 + C_\gamma.\gamma$
Yaw	$a.\left(C_{\alpha 1}.\alpha_1 + C_\gamma.\gamma\right) - b.C_{\alpha 2}.\alpha_2$
Roll	$\left(m_m.h_m + m_f.h_f\right)g.\varphi.\cos(\mu) +$ $+\left(m_f.c_1.g - k.F_{z1}\right)\delta.\cos(\mu) +$ $-h_c.\left(C_{\alpha 1}.\alpha_1 + C_\gamma.\gamma\right) + T_\varphi$
Steering	$k.\left(C_{\alpha 1}.\alpha_1 + C_\gamma.\gamma\right) + B.\dot{\delta} +$ $+\left(m_f.c_1.g - k.F_{z1}\right)\gamma + T_\delta$